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## 12th Global Conference on Sustainable Manufacturing

## Substitution in a hybrid remanufacturing system

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**Abstract**

Increasing legislative and societal pressures are requiring manufacturers to operate more sustainably and to take responsibility for the fate of their goods after they have been used by consumers. A hybrid remanufacturing system, in which newly produced and remanufactured used goods are sold on separate markets but also act substitutes for each other, is described and modelled using a semi-Markov decision process. The model provides an optimal policy, which specifies production, remanufacturing and substitution decisions. The model is used to explore the properties of this hybrid remanufacturing system, and in particular, the managerial implications associated with upward and downward substitution.

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**1. Introduction**

Remanufacturing is the process by which products are restored to an “as-new” condition [1]. Remanufacturing and more broadly, product recovery, have received an increasing amount of attention recently as governments [2,3], businesses [4,5,6] and consumers become more aware of the importance of sustainability and reducing the impact on the environment. In some industries, consumers may not differentiate between new and remanufactured products. However, this is not always the case as despite the fact that remanufactured products are typically sold with a warranty equivalent to that of a new product [1,7], consumers still have concerns about the quality of remanufactured products [1] so view them as inferior. The paper considers substitution strategies for a company selling new and remanufactured products to its customers. The model developed considers customers' preferences for new and remanufactured products and their willingness to accept a substitute. The model is used to explore the conditions under which it is optimal for the company to offer substitution.

Following the literature we define downward substitution as when a superior product is used to satisfy demand for an inferior product, and upward substitution as when an inferior product is used to satisfy demand for a superior product [8,9,

10]. Much of the research in the field of product recovery with separate markets for newly produced goods and recovered (or remanufactured) goods stems from the work of Inderfurth [9], who investigates a stochastic single period model with downward substitution. It was found that offering substitution leads to lower levels of recovered inventory. Single-period problems with two-way substitution have been studied by Bayindir et al. [11] and Kaya [12], who assume that the proportion of customers who accept a substitution is known and constant. The quality of returned used goods is addressed by Kaya [12] by way of an incentive paid to the consumer in exchange for returning used goods.

Previous research has considered models over more than one time period. Aras et al. [13] study a finite horizon, periodic review system in which new products are leased, and then returned, recovered and then sold; substitution is not offered. Ahiska and Kurtul [14] consider an infinite horizon Markov decision process model for a manufacturing/remanufacturing system in which downward substitution is offered. They found that the relative cost of remanufacturing compared with manufacturing was a key factor in determining profitability.

Deterministic models have also been used to study product recovery models with separate markets and

substitution [15,16,17,18]. The relationship between acceptance of substitution and the level of compensation paid to the consumer is also investigated [17].

The current paper extends the literature, in particular by including stochastic demand for new and remanufactured goods and return of used goods, allowing two-way substitution with uncertain consumer acceptance, including two quality dependent recovery channels, and utilizing an infinite horizon model.

## 2. Problem description and model formulation

The hybrid production and remanufacturing system modelled in this paper is presented in Figure 1. Newly produced goods and the lower priced remanufactured goods are sold on separate markets. Both types of goods are functionally identical, therefore they can act as substitutes for each other through upward and downward substitution if a stock-out occurs. Decisions about offering substitution are made at a strategic level, as well as an operational level. At a strategic level a decision is made about whether or not substitution will ever be offered, then at an operational level a decision is made about whether or not substitution will be offered to a particular customer given certain inventory levels.

The recovery process primarily remanufactures used goods to be “as new”. However, if the used goods are of poor quality, then remanufacturing may not be cost-efficient. In these cases components can be salvaged from used goods and then used in the production process. We refer to the process resulting in remanufactured goods as “high-quality recovery” and the process resulting in components as “low-quality recovery”. Additional components are bought on an ‘as-needed’ basis. It is assumed that the fixed cost for buying components is negligible and that there is a short lead time, meaning there is no need to buy and store these additional components in advance.

There are four inventories in this model: used, produced, remanufactured and component. At each decision epoch the inventory levels are assessed and decisions are made about whether to produce new goods, recover used goods or order components. Furthermore, a substitution decision is made which specifies whether or not substitution should be offered if there is a stock-out during the time between the current and the next decision epochs. Demand that is not met (either through the supply of the requested product or substitution) is lost. There are no backorders. Production and recovery occur in batches and we assume that only one batch can be outstanding at any given time. We assume that customers arrive one at a time and each demand one item. The objective is to maximum the long run average reward. This continuous-time product recovery problem is modelled using a semi-Markov decision process.

### 2.1. Decision Epochs

The system is reviewed and decisions are made after the (1) arrival of a production order, (2) arrival of a recovery order, (3) demand for produced goods, (4) demand for remanufactured goods, or (5) arrival of used goods.

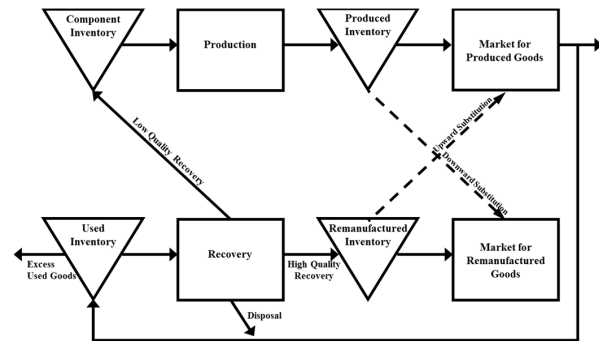


Fig. 1. Product recovery model with separate markets and substitution.

### 2.2. States

The state of the system is characterized by six state variables, including four inventory levels and two outstanding-order variables. The inventory state variables represent the levels of produced, remanufactured, used and component inventories and are denoted by  $i_p, i_r, i_u$  and  $i_c$  respectively. The inventory levels are bounded as follows:  $I_p^{min} \leq i_p \leq I_p^{max}$ ,  $I_r^{min} \leq i_r \leq I_r^{max}$ ,  $I_u^{min} \leq i_u \leq I_u^{max}$ , and  $I_c^{min} \leq i_c \leq I_c^{max}$ . Since there are no backorders, the minimum inventory levels are 0. The numbers of goods in outstanding production and recovery orders are denoted  $i_{op}$  and  $i_{or}$  respectively. These variables are bounded as follows:  $0 \leq i_{op} \leq A_p^{max}$  and  $0 \leq i_{or} \leq A_r^{max}$  where  $A_p^{max}$  and  $A_r^{max}$  are the maximum order sizes for production and recovery respectively.

### 2.3. Actions, costs and transition probabilities

At each decision epoch the state of the system is reviewed and decisions are made regarding replenishment and substitution.

#### 2.3.1. Replenishment actions.

It is assumed that a production (recovery) order can only be placed if there is not already an outstanding production (recovery) order. At each decision epoch the firm may choose to either produce or recover, or to do neither. The size of the production and recovery orders are denoted by  $a_p$  and  $a_r$  respectively. For a given state  $i$ , the possible order sizes are determined by number of used goods in stock and the available capacity in the relevant inventories. The number of components  $r$  a production order is calculated at the completion (i.e. the arrival) of the order. The action space for production and recovery orders can be defined as follows:

$$a_p \in \begin{cases} \{0, \dots, \min\{I_p^{max} - i_p, A_p^{max}\}\}, & \text{if } i_{op} = a_r = 0 \\ \{0\}, & \text{otherwise} \end{cases}$$

$$a_r \in \begin{cases} \{0, \dots, \min\{i_u, A_r^{max}\}\}, & \text{if } i_{or} = a_p = 0 \\ \{0\}, & \text{otherwise} \end{cases}$$

The number of used items which become remanufactured items and components through the recovery process depends

on the quality of the used items, and also on the capacities of the remanufactured and component inventories. Figure 2 shows the recovery process. In a recovery order of  $a_r$  used items, there are  $x_q$  items of sufficient quality to undergo high quality recovery. The number which do become remanufactured goods (through high quality recovery) is  $a_h(x_q) = \min\{I_r^{max} - i_r, x_q\}$ ; the number which are used for components (low quality recovery) is  $a_l(x_q) = \min\{I_c^{ma} - i_c, a_r - a_h(x_q)\}$ ; and the number which are disposed of is:  $a_d(x_q) = a_r - a_h(x_q) - a_l(x_q)$ .

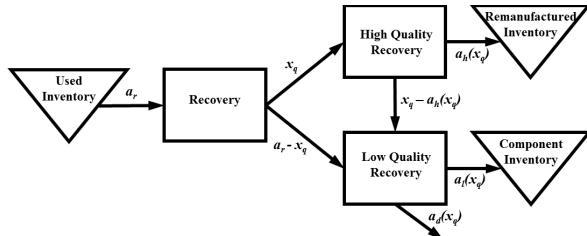


Fig. 2. Recovery of low quality and high quality used goods.

### 2.3.2. Substitution actions

At each decision epoch an operational decision is made about whether substitution will be offered should a customer demand an out-of-stock item between the current and next decision epoch. If no such customer arrives between the decision epochs then substitution will not take place. Furthermore, substitution can only take place if a customer demands a produced (remanufactured) item when there are none in stock and there is at least one remanufactured (produced) item in stock and if upward (downward) substitution is permitted at both a strategic and operational level. The strategic decision is made outwith the model. The operational decisions regarding upward and downward substitution are denoted by  $a_U$  and  $a_D$  respectively where:

$$a_k = \begin{cases} 1 & \text{if substitution is offered} \\ 0 & \text{otherwise} \end{cases} \text{ for } k = U, D.$$

Since it is assumed that each customer demands only one item, the variables  $a_U$ ,  $a_D$  also denote the number of goods offered for substitution (either 1 or 0). If offered a substitution, then the customer can choose to accept or reject the substitute item.

### 2.3.3. Transition probabilities, costs and revenues

The objective of this semi-Markov decision process is to maximize the long-run average reward (revenues less costs). Revenues are received for the sale of produced and remanufactured goods, and costs are incurred for: holding inventory, placing orders, lost sales, substitution and disposal of items. The reward evaluated at each decision epoch is the expected reward that will be received until the next decision epoch. To assist with presentation of this section, we define an indicator function  $\delta$  as follows:

$$\delta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

### 2.3.4. Time until next decision epoch

The lead times of production and recovery orders are modelled as independent exponential random variables with

means  $1/\mu_p$  and  $1/\mu_r$  respectively. However, the next event can only be the arrival of a production or recovery order if there is already an order outstanding ( $i_{op} = 1$  or  $i_{or} = 1$ ) or if a replenishment order is placed ( $a_p > 0$  or  $a_r > 0$ ). After replenishment decisions have been made, the number of outstanding produced items is  $i_{op} + a_p$  and the number of outstanding recovered items is  $i_{or} + a_r$ . Demands for produced and remanufactured goods are modelled as independent Poisson processes with rates  $\lambda_p$  and  $\lambda_r$  respectively. The arrival of used goods is modelled by a marked Poisson process with rate  $\lambda_u$ . The number of used goods returned with each "arrival" is governed by the random variable  $X_u$  with a known distribution. It follows that the time until the next decision epoch is an exponential random variable. The expected time until the next decision epoch, denoted by  $\tau_i(a)$ , can be defined as:

$$\tau_i(a) = \frac{1}{\lambda(i, a)},$$

where,

$$\lambda(i, a) = \lambda_u + \lambda_p + \lambda_r + \delta(i_{op} + a_p)\mu_p + \delta(i_{or} + a_r)\mu_r.$$

### 2.3.5. Other uncertain variables

The acceptance of upward and downward substitution is modelled by Bernoulli random variables with parameters  $\alpha_U$  and  $\alpha_D$  respectively. The number of high quality used items in a recovery order of size  $a_r$  items is modelled by a Binomial random variable,  $X_q \sim \text{Bin}(a_r, \alpha)$ .

### 2.3.6. State and action dependent costs and revenues

Holding costs of  $h_u$ ,  $h_p$ ,  $h_r$  and  $h_c$  are incurred per unit, for used, produced, remanufactured and component inventory respectively. The expected cost of holding inventory until the next decision epoch is presented in (1). Set-up costs of  $k_p$  and  $k_r$  are incurred each time a production or recovery order is placed, respectively. The set-up costs incurred until the next decision epoch are not subject to uncertainty and are presented in (2).

### 2.3.7. Transition probabilities and event dependent costs and revenues

The transition probabilities and other costs and revenues depend on the event that occurs at the next decision epoch. For clarity, let us define  $j_{op} = (i_{op} + a_p)$  as number of produced items outstanding and  $j_{or} = (i_{or} + a_r)$  as the number of remanufactured items outstanding at the end of the current decision epoch.

The probability that the next event is the arrival of a production order is  $\delta(j_{op})\mu_p/\lambda(i, a)$ . Costs of  $c_p$  and  $c_b$  are incurred on a per unit basis for each item that is produced and each component bought, respectively. The number of components required for the production order is defined as:  $a_b = \max\{0, j_{op} - i_c\}$ . The arrival of a production order results in a cost (see (3)) and transition to state:

$$j = (i_u, i_p + j_{op}, i_r, i_c + a_b - j_{op}, 0, j_{or})$$

The probability that the next event is the arrival of a recovery order is  $\delta(j_{or})\mu_r/\lambda(i, a)$ . A cost of  $c_r$  is incurred per unit recovered. In addition there are unit costs of  $c_h$ ,  $c_l$  and  $c_d$  per unit for high quality recovery, low quality recovery and disposal. When this event occurs, the cost incurred and the state

transition depend on the yield of high quality used items. The expected cost until the next decision epoch is presented in (4) and the next state  $j$  is:

$$j = (i_u, i_p, i_r + a_h(X_q), i_c + a_l(X_q), j_{op}, 0)$$

The probability that the next event is the demand for a produced or remanufactured item is given  $\lambda_p/\lambda(i, a)$  and  $\lambda_r/\lambda(i, a)$  respectively. Revenues of  $p_p$  and  $p_r$  per item are received for the sale of newly produced and remanufactured goods, respectively, ( $p_p > p_r$ ). If substitution occurs (in either direction) the customer is charged  $p_r$ , the cost of the cheaper of the two goods. If there is insufficient produced (remanufactured) inventory in stock and upward (downward) substitution is not offered then the sale is lost and a lost sales cost of  $l_p$  ( $l_r$ ) is incurred. The next state  $j$  when the next event is the arrival of demand for a produced good is shown in Table 1. Upward substitution is rejected with probability  $1 - \alpha_u$  and accepted with probability  $\alpha_u$ . The expected cost until the next decision epoch is presented in (5).

Table 1. Next state when after demand for a produced item.

Next state $j$	Condition
$(i_u, i_p - 1, i_r, i_c, j_{op}, j_{or})$	$i_p > 0$
$(i_u, i_p, i_r, i_c, j_{op}, j_{or})$	$i_p = \alpha_u = 0$
$(i_u, i_p, i_r, i_c, j_{op}, j_{or})$	$i_p = 0, \alpha_u = 1$ and substitution is rejected
$(i_u, i_p, i_r - 1, i_c, j_{op}, j_{or})$	$i_p = 0, \alpha_u = 1$ and substitution is accepted

The next state  $j$  when the next event is demand for a remanufactured item is shown in Table 2. Downward substitution is rejected with probability  $1 - \alpha_d$  and accepted with probability  $\alpha_d$ . The expected cost until the next decision epoch is presented in (6).

Table 2. Next state when after demand for a remanufactured item.

Next state $j$	Condition
$(i_u, i_p, i_r - 1, i_c, j_{op}, j_{or})$	$i_r > 0$
$(i_u, i_p, i_r, i_c, j_{op}, j_{or})$	$i_r = \alpha_d = 0$
$(i_u, i_p, i_r, i_c, j_{op}, j_{or})$	$i_r = 0, \alpha_d = 1$ and substitution is rejected
$(i_u, i_p - 1, i_r, i_c, j_{op}, j_{or})$	$i_r = 0, \alpha_d = 1$ and substitution is accepted

The probability that the next event is the arrival of used items is  $\lambda_u/\lambda(i, a)$ . A unit cost of  $c_a$  is incurred for acquiring used goods and a unit cost of  $l_u$  is incurred for disposing of any used goods that do not fit within the used goods inventory. If there are  $X_r$  used goods, then the expected cost until the next decision epoch is (7) and the next state  $j$  is:

$$j = (i_u + \min\{X_r, I_r^{max} - i_u\}, i_p, i_r, i_c, j_{op}, j_{or})$$

The expected total reward, for a given state  $i$  and action  $a$ , is:

$$\begin{aligned} E[R(i, a)] = & -E[C_h(i, a)] - C_k(i, a) - \frac{\delta(j_{op})\mu_p}{\lambda(i, a)} C_{op}(i, a) \\ & - \frac{\delta(j_{or})\mu_r}{\lambda(i, a)} E[C_{or}(i, a)] \\ & + \frac{\lambda_p}{\lambda(i, a)} E[R_p(i, a)] + \frac{\lambda_r}{\lambda(i, a)} E[R_r(i, a)] \\ & - \frac{\lambda_u}{\lambda(i, a)} E[C_u(i, a)] \end{aligned}$$

where the individual terms are:

$$E[C_h(i, a)] = (h_u i_u + h_p i_p + h_r i_r + h_c i_c) \tau_i(a) \quad (1)$$

$$C_k(i, a) = k_p \delta(a_p) + k_r \delta(a_r) \quad (2)$$

$$C_{op}(i, a) = j_{op} c_p + a_b c_b \quad (3)$$

$$E[C_{or}(i, a)] = a_r c_r + E[a_h(X_q)] c_h + E[a_l(X_q)] c_l + E[a_d(X_q)] c_d \quad (4)$$

$$E[R_p(i, a)] = \delta(i_p) p_p + (1 - \delta(i_p)) \{ (1 - \alpha_u)(-l_p) + \alpha_u [\alpha_u p_r - (1 - \alpha_u)(-l_p)] \} \quad (5)$$

$$E[R_r(i, a)] = \delta(i_r) p_r + (1 - \delta(i_r)) \{ (1 - \alpha_d)(-l_r) + \alpha_d [\alpha_d p_p - (1 - \alpha_d)(-l_r)] \} \quad (6)$$

$$E[C_u(i, a)] = E[X_u] c_u + E[\max\{0, X_u - I_u^{max} + i_u\}] l_u \quad (7)$$

### 3. Computational Study

The behaviour of the model and the implications of the four substitution strategies have been investigated through a computational study. This semi-Markov decision problem has six state variables and four action variables, therefore there is a significant computational burden associated with obtaining an optimal policy for this model. To aid with the computational investigations a simplification is proposed; it involves the introduction of fixed order sizes of  $Q_p$  for production and of  $Q_r$  for recovery. The action space for production and recovery then become:

$$\begin{aligned} a_p & \in \begin{cases} \{0, \min\{I_p^{max} - i_p, Q_p\}\}, & \text{if } i_{op} = a_r = 0 \\ \{0\}, & \text{otherwise} \end{cases} \\ a_r & \in \begin{cases} \{0, Q_r\}, & \text{if } i_{or} = a_p = 0 \text{ and } i_u \geq Q_r \\ \{0\}, & \text{otherwise} \end{cases} \end{aligned}$$

The parameter values for the problems are presented in Table 3. For all problems, the inventory levels can take discrete values from 0 to 20. For each of the 20 problems, three different order sizes for production  $Q_p$  and recovery  $Q_r$  are used to give three problem sets  $P1$ ,  $P2$ , and  $P3$ . The order sizes are based on the mean and variance of demand:

$$P1: Q_p = \lceil \lambda_p + \sqrt{\lambda_p} \rceil, \quad Q_r = \lceil \lambda_r + \sqrt{\lambda_r} \rceil$$

$$P2: Q_p = \lceil \lambda_p + 2\sqrt{\lambda_p} \rceil, \quad Q_r = \lceil \lambda_r + \sqrt{\lambda_r} \rceil$$

$$P3: Q_p = \lceil \lambda_p + \sqrt{\lambda_p} \rceil, \quad Q_r = \lceil \lambda_r + 2\sqrt{\lambda_r} \rceil$$

where the function  $\lceil x \rceil$  rounds  $x$  up to the nearest integer. The behaviour of the optimal policy under these conditions is analyzed by examining the average reward under the four substitution strategies: no substitution, upward substitution, downward substitution, two-way substitution. The value iteration algorithm is used to obtain the optimal policy.

The average rewards for the three problems sets, under the four substitution strategies were investigated. The average rewards for problem set  $P1$  are presented in Figure 3. The value of the average reward varies across the problems as well as between the substitution strategies. This is not surprising given the large variations in parameter values across the 20 problems. What is of more interest is the difference between the substitution strategies across the 20 problems. As confirmed by Figure 3, the reward associated with the two-way substitution is the highest. Note, however, that for some problems the



Table 3. Problem parameters.

#	$\lambda_u$	$\lambda_p$	$\lambda_r$	$\mu_p$	$\mu_r$	$X_u \sim U$	$\alpha$	$\alpha_U$	$\alpha_D$	$c_p$	$c_r$
1	5	6	5	30	18	(2,7)	0.8	0.68	0.44	106.5	13.5
2	3	6	1	24	15	(0,5)	0.2	0.65	0.75	66	34
3	3	8	8	16	12	(0,5)	0.3	0.57	0.9	88	42
4	3	6	5	24	12	(0,5)	0.3	1	0.62	131	89
5	6	7	6	21	14	(3,8)	0.5	1	0.81	161	29
6	6	7	2	21	14	(3,8)	0.7	0.55	0.53	107	13
7	6	7	7	18	13	(3,8)	0.8	0.5	0.5	66	44
8	6	6	4	30	15	(3,8)	0.8	0.4	0.72	75	15
9	3	6	1	24	18	(0,5)	0.2	0.75	0.74	212.5	27.5
10	6	7	4	21	14	(3,8)	0.5	0.94	0.87	188.5	141.5
11	5	9	6	13	11	(2,7)	0.5	0.92	0.87	23.5	6.5
12	5	9	2	13	11	(2,7)	0.5	0.8	0.76	76	24
13	5	9	5	11	10	(2,7)	0.5	0.63	0.93	78.5	21.5
14	4	6	2	30	18	(1,6)	0.5	0.93	0.92	106.5	13.5
15	5	6	4	24	15	(2,7)	0.4	0.5	0.7	65.5	34.5
16	1	6	6	24	36	(1,1)	0.1	0.52	0.48	110	0
17	4	6	5	24	32	(1,6)	0.6	0.86	0.48	107	13
18	2	5	2	50	10	(2,2)	0.2	0.65	0.56	75	15
19	6	6	2	24	48	(3,8)	0.2	0.98	0.89	212.5	27.5
20	2	4	2	80	20	(2,2)	0.5	0.56	0.67	160	10

#	$c_b$	$c_h$	$c_l$	$c_d$	$l_p$	$l_r$	$p_p$	$p_r$	$k_p$	$k_r$
1	30	50	9	6.5	13	5.7	209	147	20	5
2	60	30	18	6	12	5.8	169	133	10	5
3	80	40	24	8	16	7.4	269	202	10	10
4	100	60	30	11	22	13.8	316	161	20	12
5	70	75	21	11	22	9.3	434	391	20	8
6	40	50	12	7	14	5.6	269	253	20	20
7	60	30	18	6	12	6.8	242	202	30	20
8	30	35	9	5	10	4.5	112	84	30	25
9	50	100	15	12.5	25	11.5	378	361	50	30
10	200	85	60	18.5	37	20.8	563	529	28	8
11	50	10	15	3.5	7	1.3	146	83	12	2
12	50	35	15	6	12	5.3	214	119	12	2
13	100	35	30	8.5	17	4.8	277	259	120	10
14	30	50	9	6.5	13	5.7	231	221	20	5
15	50	30	15	5.5	11	5.9	186	111	100	50
16	100	50	30	10	20	4	330	236	1000	100
17	40	50	12	7	14	5.6	212	113	400	400
18	30	35	9	5	10	4.5	140	115	30	5
19	50	100	15	12.5	25	11.5	263	150	50	10
20	50	75	15	10	20	7.5	279	162	28	8

reward from the two-way substitution strategy is equaled by other substitution strategies.

Figure 3 was typical of the average rewards for problem sets  $P2$  and  $P3$ . It was found that the average reward was relatively robust across the three problems sets, with exception of problems with a very large set-up cost (e.g. #16 and #17) where

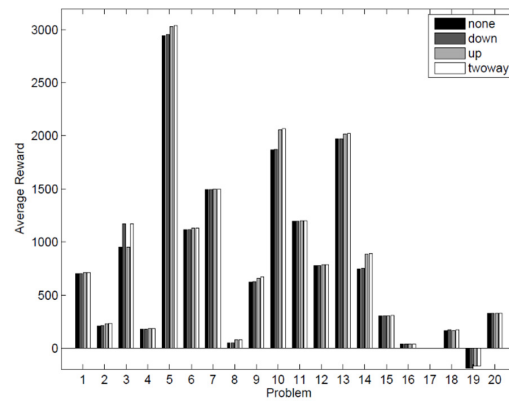


Fig. 3. Average reward of the optimal policy for all substitution strategies for problem set P1.

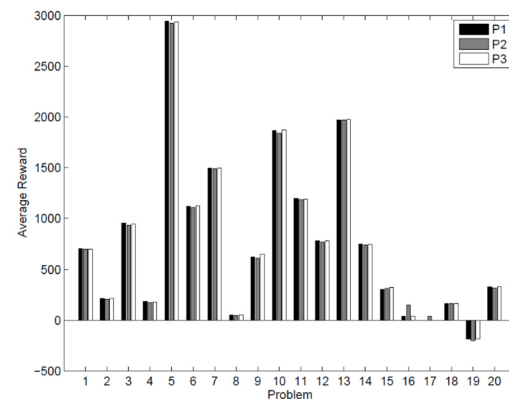


Fig. 4. Average reward of the optimal policy for all problem sets under a two-way substitution strategy.

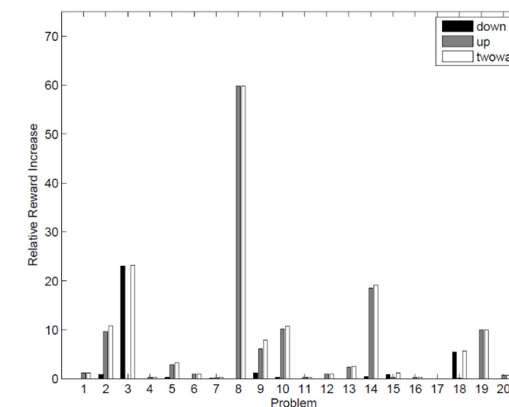


Fig. 5. Relative reward increase for substitution strategies for problem set P1.

an increase in the production order size (from  $P1$  to  $P2$ ) resulted in a large increase in the average reward. This was similar across all substitution strategies. For example under the two-way substitution strategy and the  $P1$  order sizes, #17 had an average reward of 1.08 units, whereas with the  $P2$  order sizes the average reward was 39.47 units. This is visible in

Figure 4, which shows the variation between the three problem sets for the two-way substitution strategy. Further analysis is required to examine the influence of other parameter values.

To investigate the additional reward achievable through each of the substitution strategies, the relative reward increase (RRI) from substitution is compared with not allowing substitution:  $RRI = \{[\text{Reward}(\text{substitution}) - \text{Reward}(\text{no substitution})] / \text{Reward}(\text{no substitution}) \times 100\%$ . Figure 5 which shows the relative reward increase for problem set *P1* reveals that the additional reward available by allowing two-way substitution varies considerably between the 20 problems. The RRI attainable by allowing substitution varies from 0 (no benefit from substitution) to approximately 60% (substantial benefit from substitution). For some problems (e.g. #08) the increase in reward available by allowing upward substitution is greater than for downward substitution, however for some problems (e.g. #03) the reverse can be observed. This variation suggests that the benefit available from allowing substitution depends heavily on the problem parameters. In general, the greatest increase in reward is attained by allowing a substitution strategy which includes upward substitution. These numerical results highlight some interesting properties of the model, however further work, including a sensitive analysis, is also required to obtain deeper insight into benefits attainable through substitution.

#### 4. Managerial Implications and Conclusion

This paper has proposed and analysed a continuous-time product recovery model with separate markets and substitution. Four substitution strategies were investigated. The key finding of this research is that offering substitution between produced and remanufactured goods provides an increase in the long-run average reward. The size of this increase is dependent on the costs and other parameters values. By offering substitution the firm is able gain some revenue rather than losing sales and goodwill of consumers. However, there are some trade-offs associated with substitution. In offering downward (upward) substitution the firm faces the risk that the produced (recovered) inventory may also run out before the next replenishment arrives. However, these risks are not symmetric. With regard to downward substitution the firm must therefore consider the following trade-off: not offering substitution and losing sales of remanufactured goods; or offering substitution, selling produced goods for a lower price and potentially losing future sales of produced goods. Upward substitution, on the other hand, involves a trade-off only with respect to potential for future lost sales, not with respect to the revenue received. Upward substitution could be seen to carry less risk than downward substitution.

The findings of this paper also highlight the need for a more comprehensive computational study and sensitivity analysis to explore in more detail the situations in which substitution can provide benefits to a hybrid producer/remanufacturer. This is an area for future research. This study revealed that the optimal policy has a very complicated structure, which would be

impractical to implement in industry. Therefore future research could also investigate the structure of the optimal policy in order to gain insights which could be used to inform the development of simple, yet effective, policies for use by managers in charge of such systems.

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